Appendix E
Operational Amplifiers

TO ACCOMPANY

AUTOMATIC CONTROL SYSTEMS
EIGHTH EDITION

BY

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JOHN WILEY & SONS, INC.

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ISBN 0-471-13476-7

APPENDIX **E**

Operational Amplifiers

E-1 OPERATIONAL AMPLIFIERS

Operational amplifiers, or simply **op-amps**, offer a convenient way to build, implement, or realize continuous-data or *s*-domain transfer functions. In control systems, op-amps are often used to implement the controllers or compensators that evolve from the control-system design process, so in this appendix we illustrate common op-amp configurations. An in-depth presentation of op-amps is beyond the scope of this text. For those interested, many texts are available that are devoted to all aspects of op-amp circuit design and applications [References 1 and 2].

Our primary goal here is to show how to implement first-order transfer functions with op-amps, while keeping in mind that higher-order transfer functions are also important. In fact, simple high-order transfer functions can be implemented by connecting first-order op-amp configurations together. Only a representative sample of the multitude of op-amp configurations will be discussed.

Some of the practical issues associated with op-amps are demonstrated in simlab (see Chapter 11).

E-1-1 The Ideal Op-Amp

When good engineering practice is used, an op-amp circuit can be accurately analyzed by considering the op-amp to be ideal. The ideal op-amp circuit is shown in Fig. E-1, and has the following properties:

1. The voltage between the + and − terminals is zero, that is, $e^+ = e^-$. This property is commonly called the *virtual ground or virtual short.*
2. The currents into the + and − input terminals are zero. Thus, the input impedance is infinite.
3. The impedance seen looking into the output terminal is zero. Thus, the output is an ideal voltage source.
4. The input-output relationship is $e_o = A(e^+ - e^-)$ where the gain A approaches infinity.

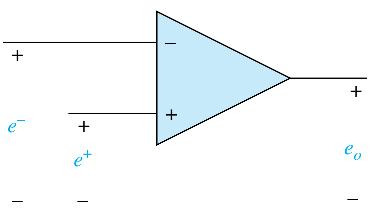

Figure E-1 Schematic diagram of an op-amp.

The input-output relationship for many op-amp configurations can be determined by using these principles. An op-amp cannot be used as shown in Fig. E-1. Rather, linear operation requires the addition of feedback of the output signal to the − input terminal.

E-1-2 Sums and Differences

As illustrated in the last chapter, one of the most fundamental elements in a block diagram or an SFG is the addition or subtraction of signals. When these signals are voltages, op-amps provide a simple way to add or subtract signals as shown in Fig. E-2,

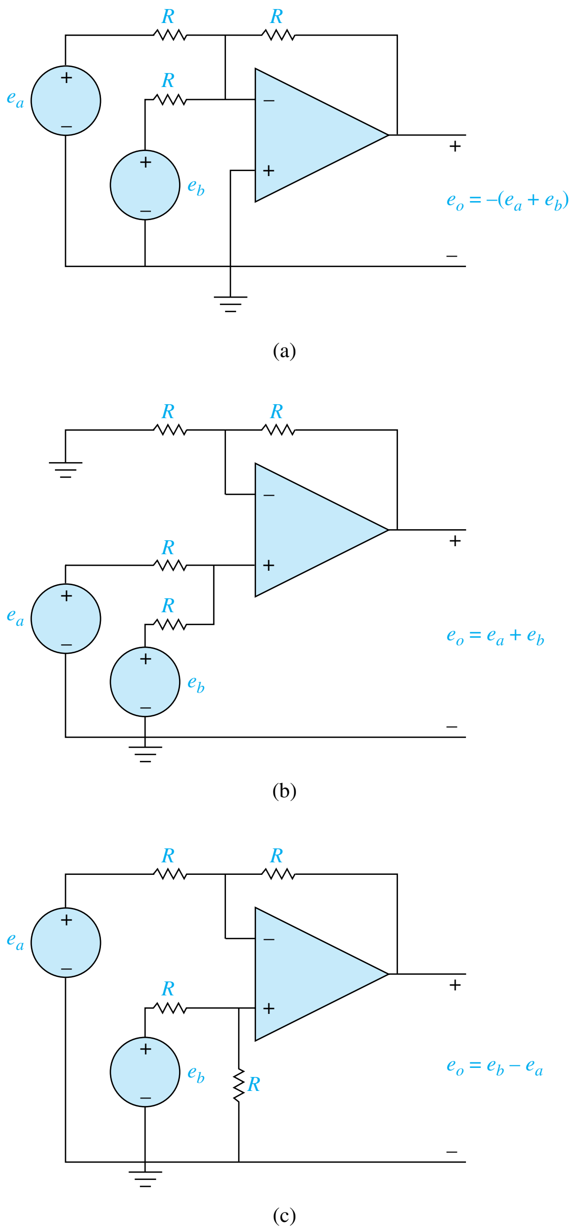

Figure E-2 Op-amps used to add and subtract signals.

where all the resistors have the same value. Using superposition and the ideal properties given in the preceding section, the input-output relationship in Fig. E-2(a) is $v_o = -(v_a - v_b)$. Thus, the output is the negative sum of the input voltages. When a positive sum is desired, the circuit shown in Fig. E-2(b) can be used. Here the output is given by $e_o = e_a + e_b$.

Modifying Fig. E-2(b) slightly gives the differencing circuit shown in Fig. E-2(c), which has an input-output relationship of $e_o = e_b - e_a$.

E-1-3 First-Order Op-Amp Configurations

In addition to adding and subtracting signals, op-amps can be used to implement transfer functions of continuous-data systems. While many alternatives are available, we will explore only those that use the inverting op-amp configuration shown in Fig. E-3. In the figure, $Z_1(s)$ and $Z_2(s)$ are impedances commonly composed of resistors and capacitors. Inductors are not commonly used because they tend to be bulkier and more expensive. Using ideal op-amp properties, the input-output relationship, or transfer function, of the circuit shown in Fig. E-3 can be written in a number of ways, such as

$$G(s) = \frac{E_o(s)}{E_i(s)} = -\frac{Z_2(s)}{Z_1(s)} = \frac{-1}{Z_1(s)Y_2(s)} \tag{E-1}$$

$$= -Z_2(s)Y_1(s) = -\frac{Y_1(s)}{Y_2(s)}$$

where $Y_1(s) = 1/Z_1(s)$ and $Y_2(s) = 1/Z(s)$ are the admittances associated with the circuit impedances. The different transfer function forms given in Eq. (E-1) apply conveniently to the different compositions of the circuit impedances.

Using the inverting op-amp configuration shown in Fig. E-3 and using resistors and capacitors as elements to compose $Z_1(s)$, and $Z_2(s)$, Table E-1 illustrates a number of common transfer function implementations. As shown in the Table E-1, it is possible to implement poles and zeros along the negative real axis as well as at the origin in the s-plane. Because the inverting op-amp configuration was used, all the transfer functions have negative gains. The negative gain is usually not an issue, since it is simple to add a gain of -1 to the input and output signal to make the net gain positive.

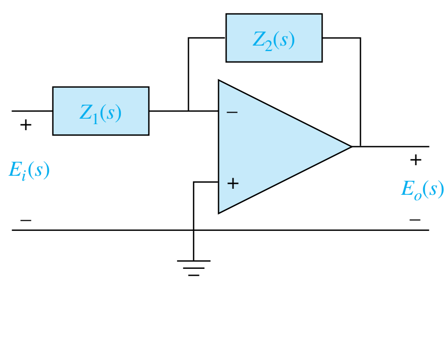

Figure E-3 Inverting op-amp configuration.

TABLE E-1 Inverting Op-Amp Transfer Functions

	Input Element	Feedback Element	Transfer Function	Comments
(a)	R_1 $Z_1 = R_1$	R_2 $Z_2 = R_2$	$-\dfrac{R_2}{R_1}$	Inverting gain. e.g., if $R_1 = R_2$, $e_o = -e_1$.
(b)	R_1 $Z_1 = R_1$	C_2 $Y_2 = sC_2$	$\left(\dfrac{-1}{R_1C_2}\right)\dfrac{1}{s}$	Pole at the origin. i.e., an integrator.
(c)	C_1 $Y_1 = sC_1$	R_2 $Z_2 = R_2$	$(-R_2C_1)s$	Zero at the origin. i.e., a differentiator.
(d)	R_1 $Z_1 = R_1$	R_2, C_2 $Y_2 = \dfrac{1}{R_2} + sC_2$	$\dfrac{-\dfrac{1}{R_1C_2}}{s + \dfrac{1}{R_2C_2}}$	Pole at $\dfrac{-1}{R_2C_2}$ with a dc gain of $-R_2/R_1$.
(e)	R_1 $Z_1 = R_1$	R_2, C_2 $Z_2 = R_2 + \dfrac{1}{sC_2}$	$\dfrac{-R_2}{R_1}\left(\dfrac{s + 1/R_2C_2}{s}\right)$	Pole at the origin and a zero at $-1/R_2C_2$, i.e., a PI Controller.
(f)	R_1, C_1 $Y_1 = \dfrac{1}{R_1} + sC_1$	R_2 $Z_2 = R_2$	$-R_2C_1\left(s + \dfrac{1}{R_1C_1}\right)$	Zero at $s = \dfrac{-1}{R_1C_1}$, i.e., a PD controller.
(g)	R_1, C_1 $Y_1 = \dfrac{1}{R_1} + sC_1$	R_2, C_2 $Y_2 = \dfrac{1}{R_2} + sC_2$	$\dfrac{\dfrac{-C_1}{C_2}\left(s + \dfrac{1}{R_1C_1}\right)}{s + \dfrac{1}{R_2C_2}}$	Pole at $s = \dfrac{-1}{R_2C_2}$ and a zero at $s = \dfrac{-1}{R_1C_1}$, i.e., a lead or lag controller.

▶ EXAMPLE E-1 As an example of op-amp realization of transfer functions, consider the transfer function

$$G(s) = K_p + \frac{K_I}{s} + K_D s \tag{E-2}$$

where K_P, K_D, and K_I are real constants. In Chapter 10 this transfer function will be called the **PID controller**, since the first term is a **P**roportional gain, the second an **I**ntegral term, and the third a **D**erivative term. Using Table E-1, the proportional gain can be implemented using line (a), the integral term can be implemented using line (b), and the derivative term can be implemented using line (c). By superposition, the output of $G(s)$ is the sum of the responses due to each term in $G(s)$. This sum can be implemented by adding an additional input resistance to the circuit shown in Fig. E-2(a). By making the sum negative, the negative gains of the proportional, integral, and derivative

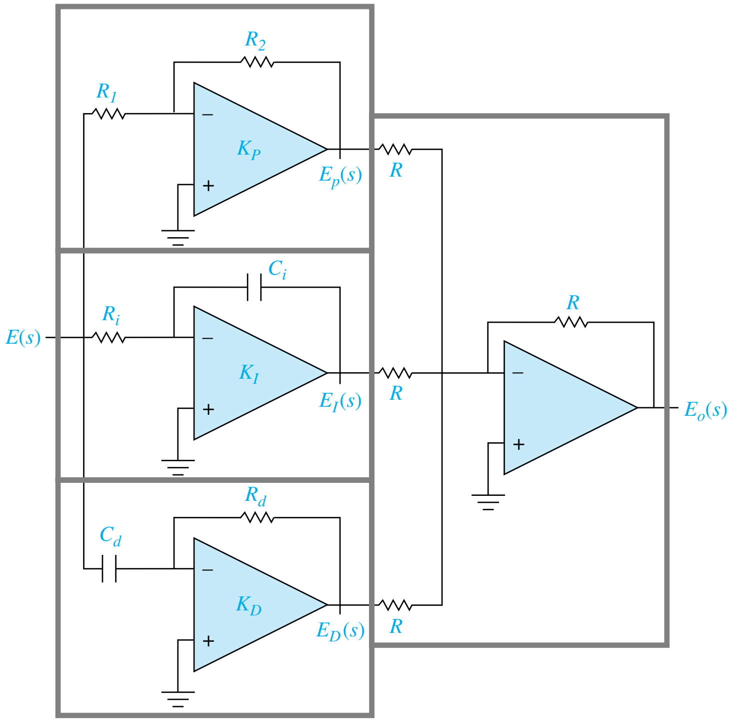

Figure E-4 Implementation of a PID controller.

term implementations are canceled, giving the desired result shown in Fig. E-4. The transfer functions of the components of the op-amp circuit in Fig. E-4 are

Proportional: $$\frac{E_P(s)}{E(s)} = -\frac{R_2}{R_1} \quad \text{(E-3)}$$

Integral: $$\frac{E_I(s)}{E(s)} = -\frac{1}{R_iC_is} \quad \text{(E-4)}$$

Derivative: $$\frac{E_D(s)}{E(s)} = -R_dC_ds \quad \text{(E-5)}$$

The output voltage is

$$E_o(s) = -[E_P(s) + E_I(s) + E_D(s)] \quad \text{(E-6)}$$

Thus, the transfer function of the PID op-amp circuit is

$$G(s) = \frac{E_o(s)}{E(s)} = \frac{R_2}{R_1} + \frac{1}{R_iC_is} + R_dC_ds \quad \text{(E-7)}$$

By equating Eqs. (E-2) and (E-7), the design is completed by choosing the values of the resistors and the capacitors of the op-amp circuit so that the desired values of K_P, K_I, and K_D are matched. The design of the controller should be guided by the availability of standard capacitors and resistors.

It is important to note that Fig. E-4 is just one of many possible implementations of Eq. (E-2). For example, it is possible to implement the PID controller with just three op-amps. Also, it is common to add components to limit the high-frequency gain of the differentiator and to limit the integrator output magnitude, which is often referred to as *antiwindup* protection. One advantage of the implementation shown in Fig. E-4 is that each of the three constants K_P, K_I, and K_D can be adjusted or tuned individually by varying resistor values in their respective op-amp circuits.

Op-amps are also used in control systems for A/D and D/A converters, sampling devices, and realization of nonlinear elements for system compensation. ◀

▶ REFERENCES

1. E. J. Kennedy, *Operational Amplifier Circuits*, Holt, Rinehart and Winston, Fort Worth, TX, 1988.
2. J. V. Wait, L. P., Huelsman, and G. A. Korn, *Introduction to Operational Amplifier Theory and Applications*, Second Edition, McGraw-Hill, New York, 1992.

▶ PROBLEM

E-1. Find the transfer functions $E_o(s)/E(s)$ for the circuits shown in Fig. EP-1.

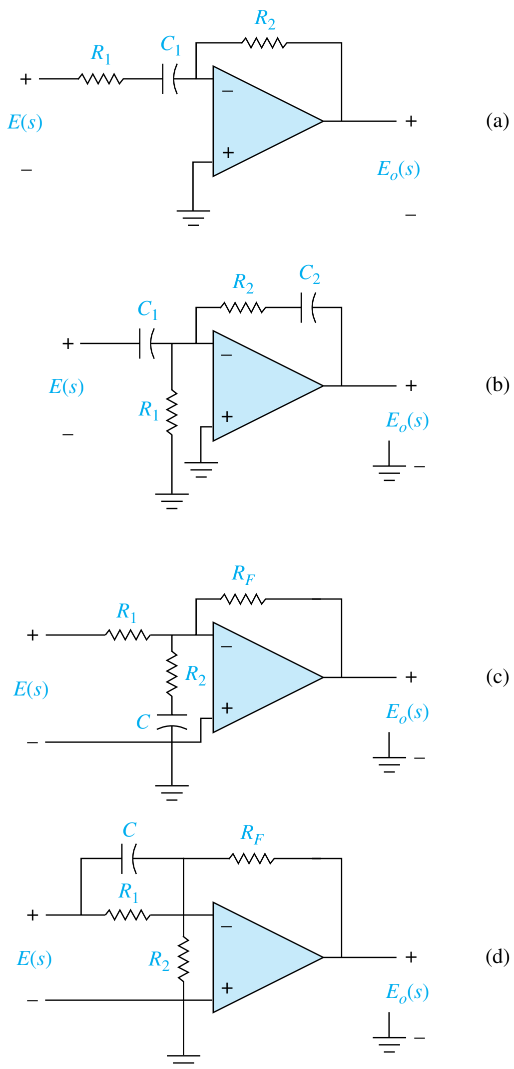

Figure EP-1